Forbidden detour move and Jones polynomials

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結び目の数学 IX

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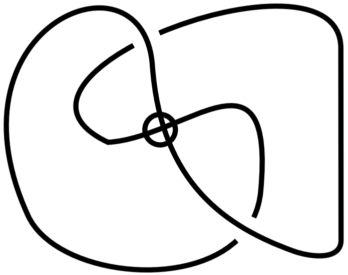
Nihon University

Virtual knot

Definition (virtual knot)

A *virtual knot* is an equivalence class of virtual knot diagrams under Reidemeister moves and virtual Reidemeister moves.



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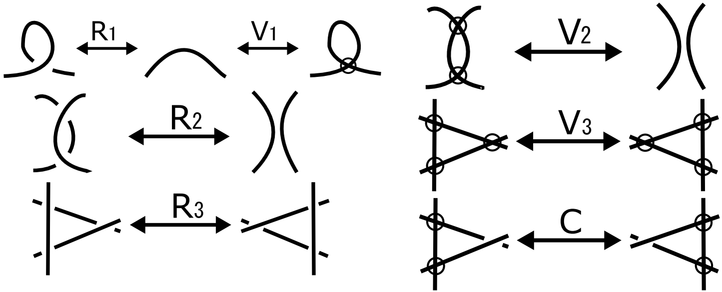
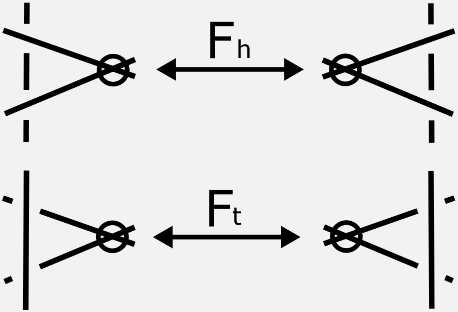


Forbidden move

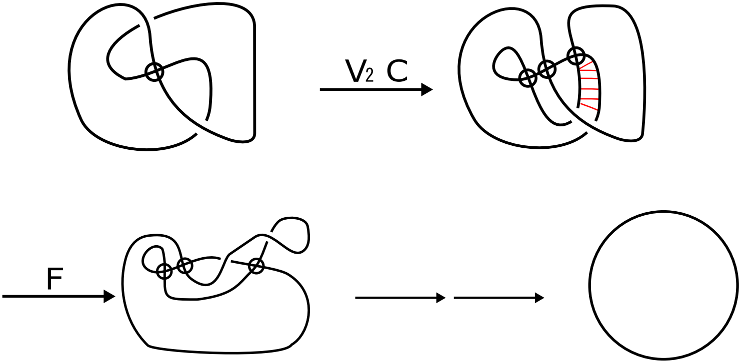
Definition (forbidden move)



Why "forbidden" ? (1)

Theorem [Kanenobu, 2001][Nelson, 2001]

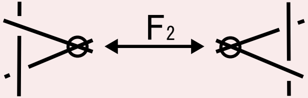
For any diagram D of a virtual knot K, there exists a finite sequence of Reidemeister moves, virtual Reidemeister moves and forbidden moves that takes D to a trivial knot diagram.



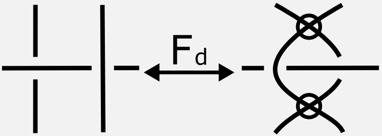
Forbidden detour move

Theorem [Kanenobu, 2001][Nelson, 2001]

An F_2-move can be realized by twice forbidden moves and some classical and virtual Reidemeister moves.



Definition (Forbidden detour move)[c.f. Crans-Meller-Ganzell]

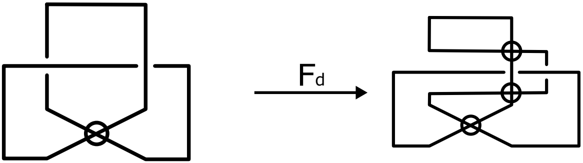


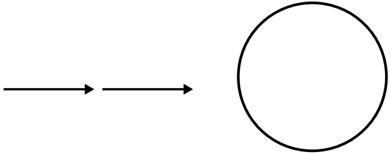
Remark F_d-move can be realized by a V_2-move and an F_2-move.

Why "forbidden" ? (2)

Theorem [Y.]

For any diagram D of a virtual knot K, there exists a finite sequence of Reidemeister moves, virtual Reidemeister moves and forbidden detour moves that takes D to a trivial knot diagram.



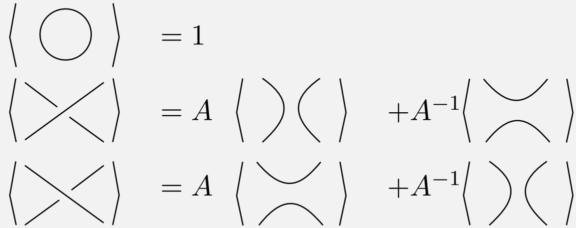


Bracket polynomial and Jones polynomial

Definition (Bracket polynomial)

Rule 1:

Rule 2:



Rule 3: $\langle D \cup \bigcirc \rangle = (-A^2 - A^{-2}) \langle D \rangle = d \langle D \rangle$

Definition (Jones polynomial)

Let K be a virtual knot and w a writhe of a diagram D of K.

$$f_K(A) = (-A^3)^{-w} \langle D \rangle$$

Remark: $V_K(t) = (-t^{-3/4})^{-w(D)} \langle D \rangle$, $A = t^{-1/4}$

Forbidden move and Jones polynomials

Theorem [Ganzell, 2014]

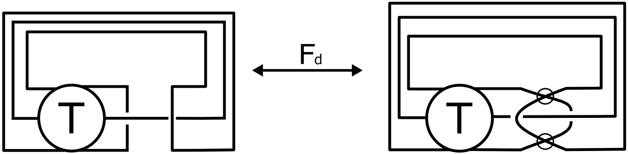
Let D and D' be diagrams of virtual knots K and K', respectively. If D and D' can be transformed into each other by a single forbidden move, then $f_K(A) - f_{K'}(A)$ is divisible by $A^{10} - A^6 - A^4 + 1$.

Theorem [Y.]

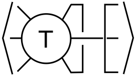
Let D and D' be diagrams of virtual knots K and K', respectively. If D and D' can be transformed into each other by a single forbidden detour move, then $f_K(A) - f_{K'}(A)$ is divisible by $A^{10} - A^6 - A^4 + 1$.

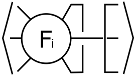
Remark: The devisors of two theorems are same,
but quotients are different.

Outline of proof (1)

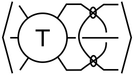
$D =$ 

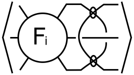
$= D'$

$\Big\langle$ 

$\Big\rangle = \sum_{i=1}^{15} p_i \Big\langle$ 

$\Big\rangle$

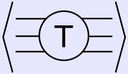
$\Big\langle$ 

$\Big\rangle = \sum_{i=1}^{15} p_i \Big\langle$ 

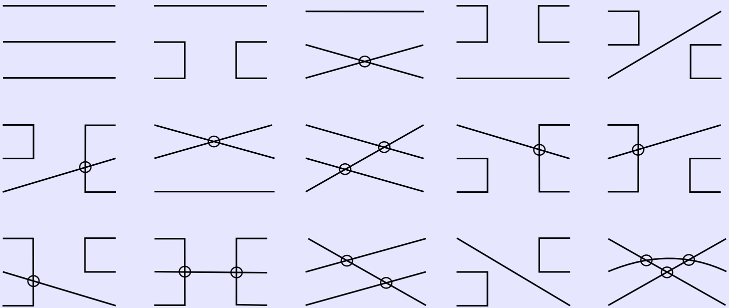
$\Big\rangle$

Outline of proof (2)

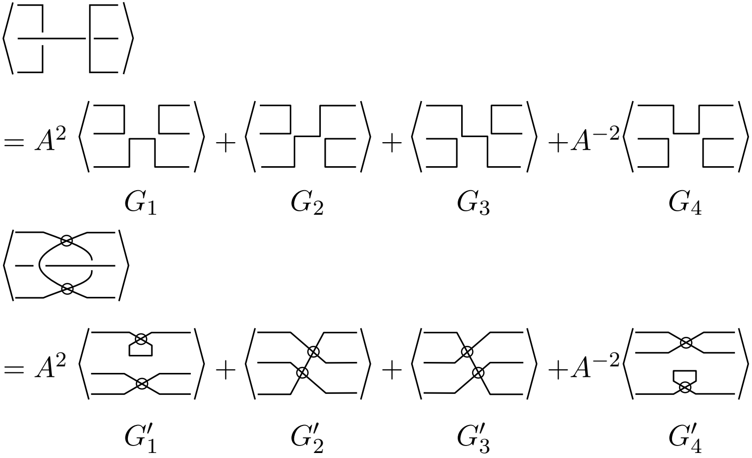
Lemma [cf. Ganzell, 2014]



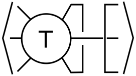
$$= \sum_{i=1}^{15} p_i \langle F_i \rangle$$



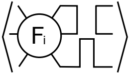
Outline of proof (3)



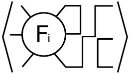
Outline of proof (4)



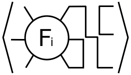
$= A^2 \sum_{i=1}^{15} p_i$



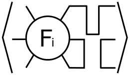
$+ \sum_{i=1}^{15} p_i$

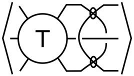


$+ \sum_{i=1}^{15} p_i$

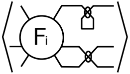


$+ A^{-2} \sum_{i=1}^{15} p_i$

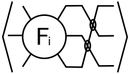




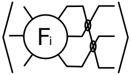
$= A^2 \sum_{i=1}^{15} p_i$



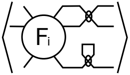
$+ \sum_{i=1}^{15} p_i$



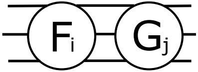
$+ \sum_{i=1}^{15} p_i$

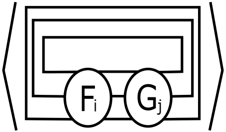


$+ A^{-2} \sum_{i=1}^{15} p_i$

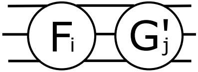


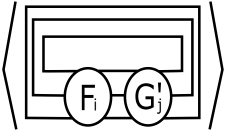
Outline of proof (5)

$\alpha_{i,j} =$ 



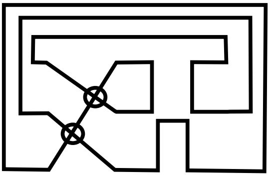
$= \langle \widehat{\alpha_{i,j}} \rangle$

$\beta_{i,j} =$ 

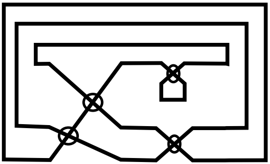


$= \langle \widehat{\beta_{i,j}} \rangle$

Outline of proof (6)

$$\langle \widehat{\alpha_{8,1}} \rangle = \Big\langle$$  $$\Big\rangle$$

$$= \langle \bigcirc \rangle = 1$$

$$\langle \widehat{\beta_{8,1}} \rangle = \Big\langle$$  $$\Big\rangle$$

$$= \langle \bigcirc \bigcirc \rangle = d$$

Outline of proof (7)

$$\begin{aligned}\langle D\rangle = \quad & p_1(A^2d+1+1+A^{-2}d)\\ &+p_2(A^2+d+d+A^{-2}d^2)\\ &+p_3(A^2+1+1+A^{-2}d)\\ &+p_4(A^2d^2+d+d+A^{-2})\\ &+p_5(A^2d+1+d^2+A^{-2}d)\\ &+p_6(A^2d+1+d+A^{-2})\\ &+p_7(A^2d+1+1+A^{-2})\\ &+p_8(A^2+1+d+A^{-2})\\ &+p_9(A^2+d+1+A^{-2}d)\\ &+p_{10}(A^2+1+d+A^{-2}d)\\ &+p_{11}(A^2d+d+1+A^{-2})\\ &+p_{12}(A^2+1+1+A^{-2})\\ &+p_{13}(A^2+d+1+A^{-2})\\ &+p_{14}(A^2d+d^2+1+A^{-2}d)\\ &+p_{15}(A^2+d+d+A^{-2})\end{aligned}$$

$$\begin{aligned}\langle D'\rangle = \quad & p_1(A^2d+1+1+A^{-2}d)\\ &+p_2(A^2d+1+1+A^{-2})\\ &+p_3(A^2d^2+d+d+A^{-2})\\ &+p_4(A^2+1+1+A^{-2}d)\\ &+p_5(A^2+1+d+A^{-2})\\ &+p_6(A^2d+d+1+A^{-2})\\ &+p_7(A^2d+d+d+A^{-2}d^2)\\ &+p_8(A^2d+1+d^2+A^{-2}d)\\ &+p_9(A^2+1+d+A^{-2}d)\\ &+p_{10}(A^2+d+1+A^{-2}d)\\ &+p_{11}(A^2d+1+d+A^{-2})\\ &+p_{12}(A^2+1+1+A^{-2})\\ &+p_{13}(A^2d+d^2+1+A^{-2}d)\\ &+p_{14}(A^2+d+1+A^{-2})\\ &+p_{15}(A^2+d+d+A^{-2})\end{aligned}$$

Outline of proof (7)

$$\langle D\rangle = p_1(A^2d+1+1+A^{-2}d) + p_2(A^2+d+d+A^{-2}d^2) + p_3(A^2+1+1+A^{-2}d) + p_4(A^2d^2+d+d+A^{-2}) + p_5(A^2d+1+d^2+A^{-2}d) + p_6(A^2d+1+d+A^{-2}) + p_7(A^2d+1+1+A^{-2}) + p_8(A^2+1+d+A^{-2}) + p_9(A^2+d+1+A^{-2}d) + p_{10}(A^2+1+d+A^{-2}d) + p_{11}(A^2d+d+1+A^{-2}) + p_{12}(A^2+1+1+A^{-2}) + p_{13}(A^2+d+1+A^{-2}) + p_{14}(A^2d+d^2+1+A^{-2}d) + p_{15}(A^2+d+d+A^{-2})$$

$$\langle D'\rangle = p_1(A^2d+1+1+A^{-2}d) + p_2(A^2d+1+1+A^{-2}) + p_3(A^2d^2+d+d+A^{-2}) + p_4(A^2+1+1+A^{-2}d) + p_5(A^2+1+d+A^{-2}) + p_6(A^2d+d+1+A^{-2}) + p_7(A^2d+d+d+A^{-2}d^2) + p_8(A^2d+1+d^2+A^{-2}d) + p_9(A^2+1+d+A^{-2}d) + p_{10}(A^2+d+1+A^{-2}d) + p_{11}(A^2d+1+d+A^{-2}) + p_{12}(A^2+1+1+A^{-2}) + p_{13}(A^2d+d^2+1+A^{-2}d) + p_{14}(A^2+d+1+A^{-2}) + p_{15}(A^2+d+d+A^{-2})$$

Outline of proof (8)

Since the writhe of D equals the writhe of D', we have

$$
\begin{aligned}
& f_K(A) - f_{K'}(A) \\
& = (-A^3)^{-w}\{p_2(A^4 - 1 - A^{-2} + A^{-6}) \\
& \quad + p_3(-A^6 + A^2 + 1 - A^{-4}) \\
& \quad + p_4(A^6 - A^2 - 1 + A^{-4}) \\
& \quad + p_7(-A^4 + 1 + A^{-2} - A^{-6})\} \\
= & (-A^3)^{-w}(p_2A^{-6} - p_3A^{-4} + p_4A^{-4} - p_7A^{-6})(A^{10} - A^6 - A^4 + 1)
\end{aligned}
$$

□