Veering triangulations of mapping tori of some pseudo-Anosov maps arising from Penner's construction

阪田 直樹

広島大学

JSPS Research Fellow (PD)

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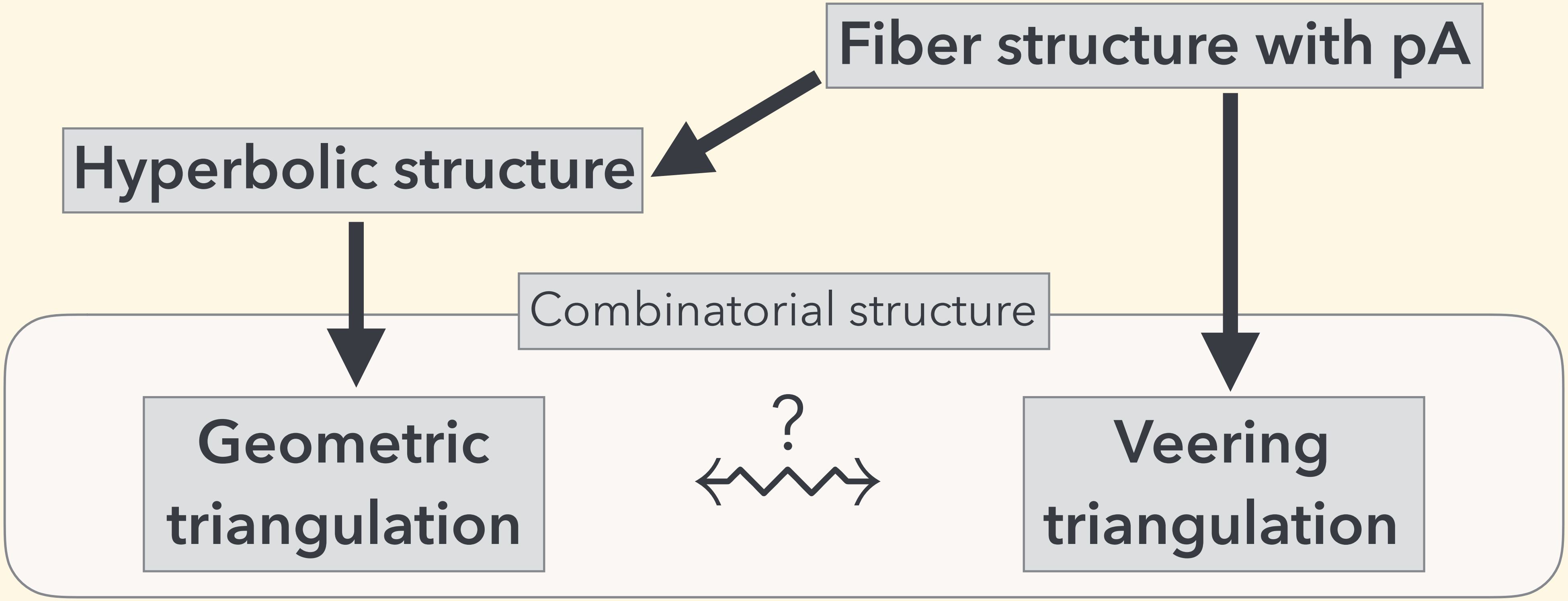
結び目の数学IX @ 日本大学

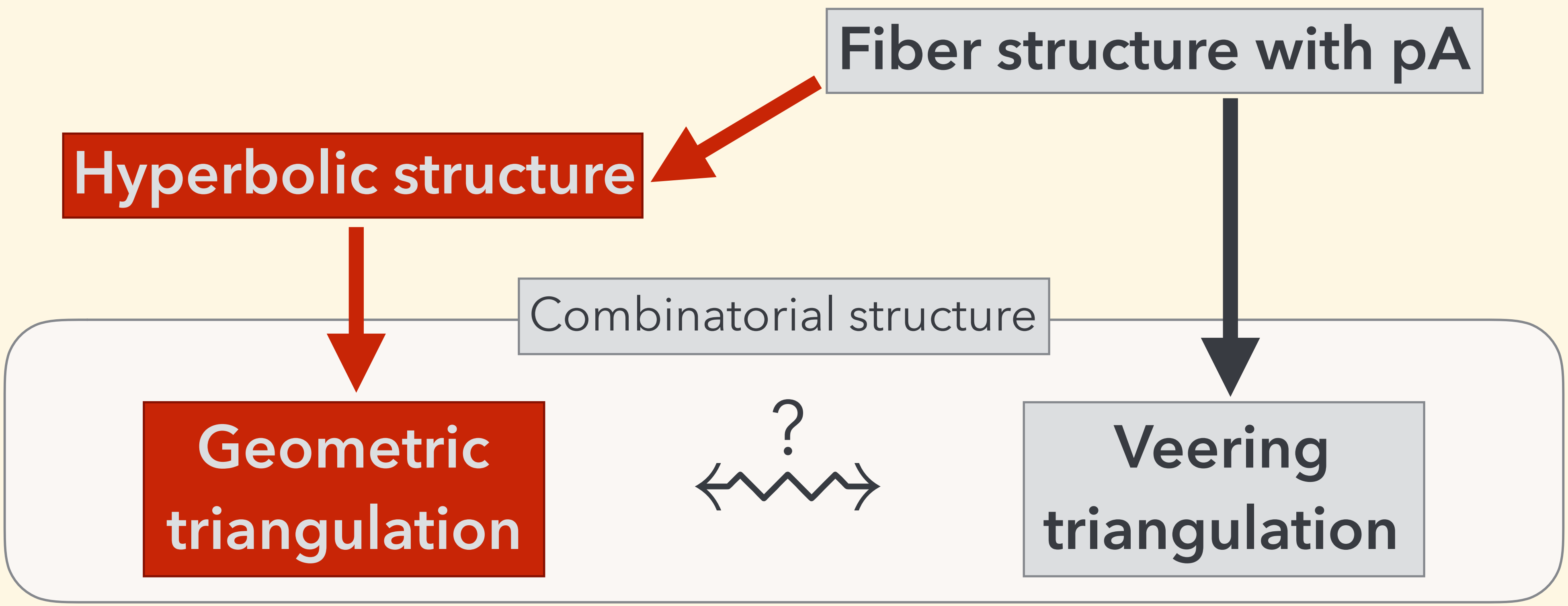
In this talk, we consider **ideal triangulations** of the mapping tori of pseudo-Anosov mapping classes.



In this talk, we consider ideal triangulations of the mapping tori of pseudo-Anosov mapping classes.

Theorem ([Epstein-Penner, 1988])

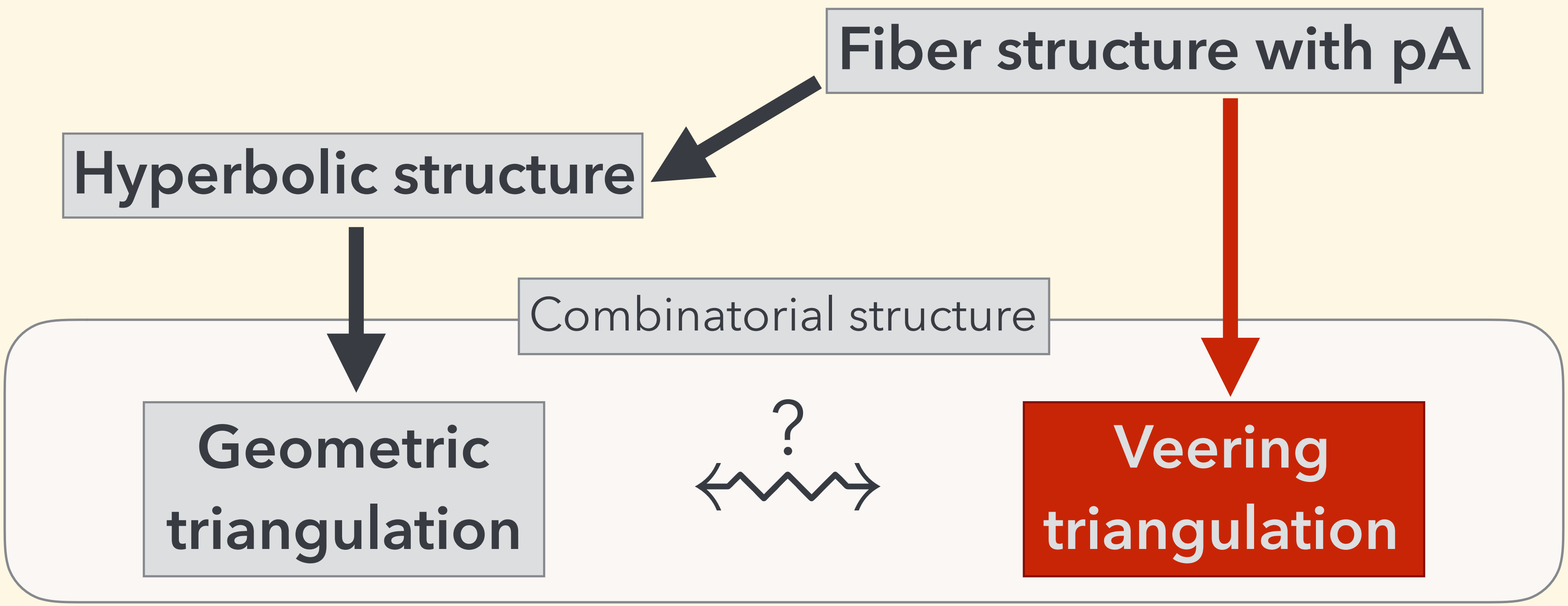
Each cusped hyperbolic manifold of finite volume admits a **canonical** decomposition into ideal polyhedra.



In this talk, we consider ideal triangulations of the mapping tori of pseudo-Anosov mapping classes.

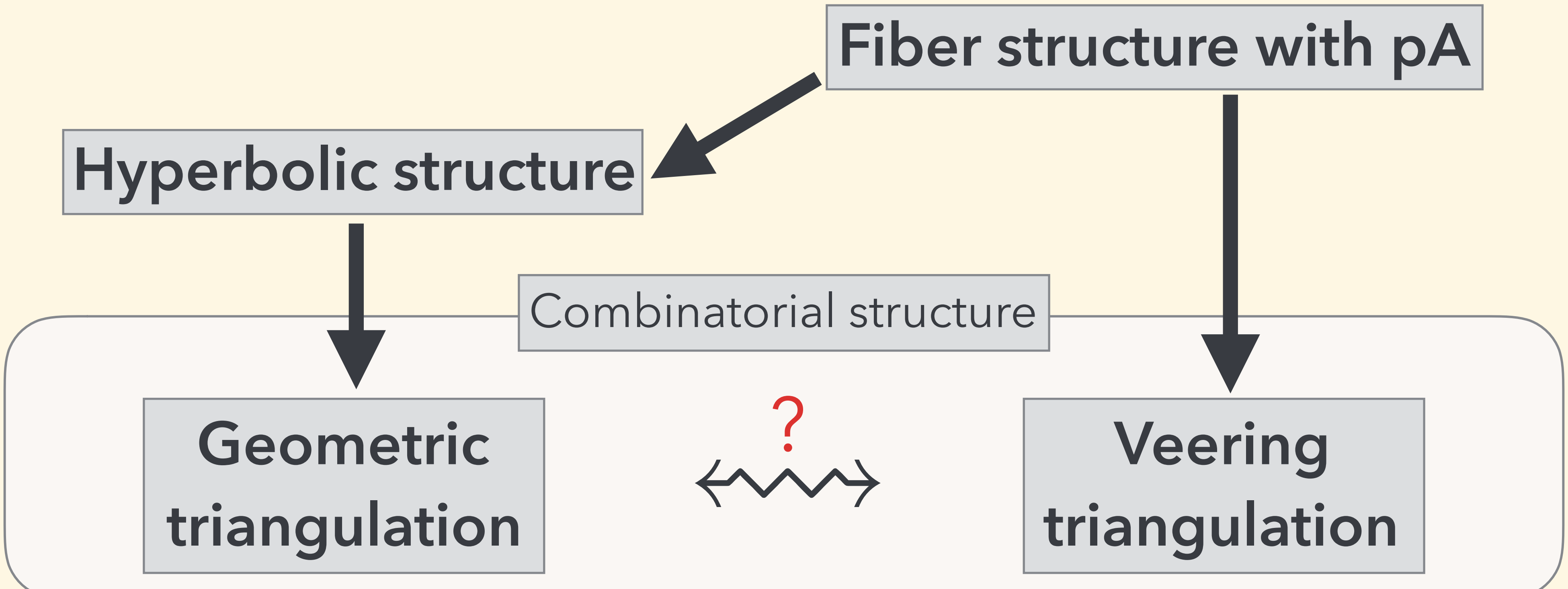
Theorem ([Agol, 2011])

For each pseudo-Anosov surface bundle over S^1, drilled along singular points of the stable/unstable foliations, has a **canonical** "veering" ideal triangulation.



Question

Is there a relationship between them?

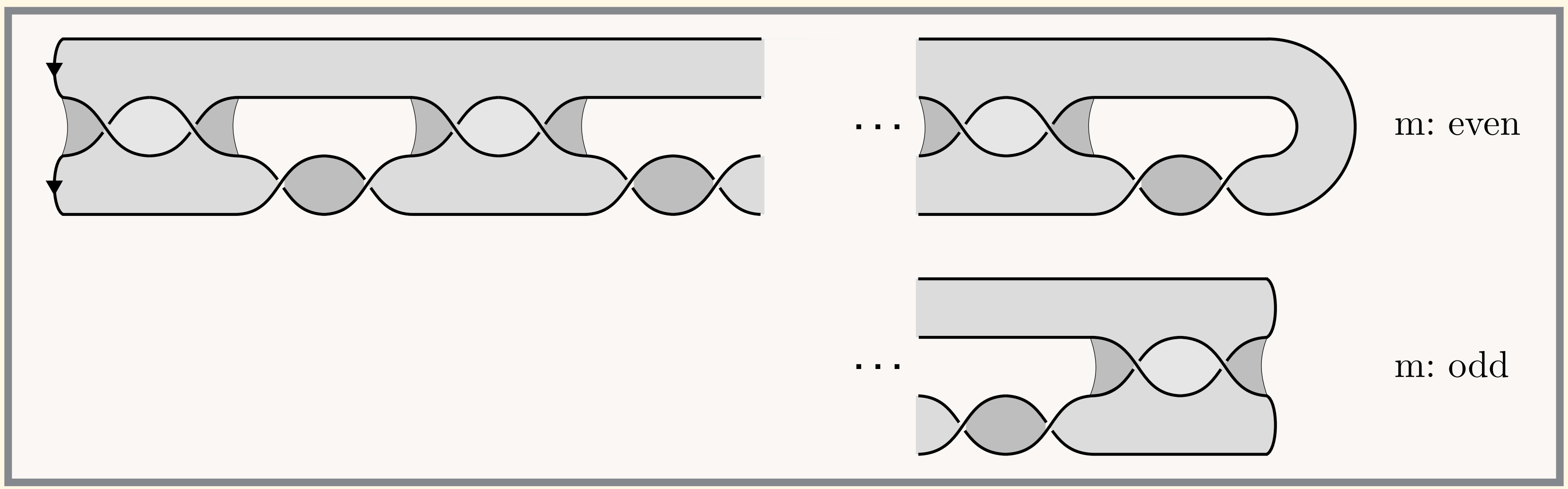


Fact (Jorgensen, etc.)

The Epstein-Penner decomposition of each **once-punctured torus bundle** over S^1 is veering.

Theorem ([S., 2016])

The Epstein-Penner decomposition of a hyperbolic fibered two-bridge link $K(r)$ $(0 < |r| < 1/2)$ is veering $\Longleftrightarrow$ the slope r has the continued fraction expansion $\pm[2, 2, \ldots, 2]$



Question

Are the veering ideal triangulations geometric?

Theorem **([Hodgson-Rubinstein-Segerman-Tillmann, 2011])**

Each veering triangulation admits a strict angle structure.

Question

Are the veering ideal triangulations geometric?

Theorem **([Hodgson-Rubinstein-Segerman-Tillmann, 2011])**

Each veering triangulation admits a strict angle structure.

However…

Theorem **([Hodgson-Issa-Segerman, 2016])**

$\exists$ a non-geometric veering ideal triangulation.

$\rightsquigarrow$ How can we characterize the geometric veering triangulations?

Definition ([Shin-Strenner, 2015])

A pseudo-Anosov mapping class is **coronal**
$\overset{\text{def}}{\iff}$ the stretch factor has a Galois conjugate on the unit circle

Theorem ([Shin-Strenner, 2015])

A coronal pseudo-Anosov mapping class has no power coming from Penner's construction.

Computer Experiment (S.)

The pseudo-Anosov mapping classes in the list of non-geometric veering triangulations contained in [Hodgson-Issa-Segerman, 2016] are coronal.

φ : pseudo-Anosov map of a surface F

$$F^\circ := F \setminus \{\text{a singular point of the stable/unstable foliation}\}$$

$$\varphi^\circ := \varphi|_{F^\circ}$$

$$M_{\varphi^\circ} := F^\circ \times [0,1]/(x,0) \sim (\varphi^\circ(x),1)$$

: the mapping torus of φ°

(In this talk, we call M_{φ° the mapping torus of φ.)

Question

Is there a relationship between "geometric veering triangulation of M_{φ°" and "coronality of φ"?

Main Question

Is the veering triangulation of the mapping torus of each pA mapping class arising from Penner's construction geometric?

Penner's construction

$\mathcal{A} := \{\alpha_1, \alpha_2, \ldots, \alpha_k\}$, $\mathcal{B} := \{\beta_1, \beta_2, \ldots, \beta_\ell\}$: mutually disjoint essential simple closed curves in F

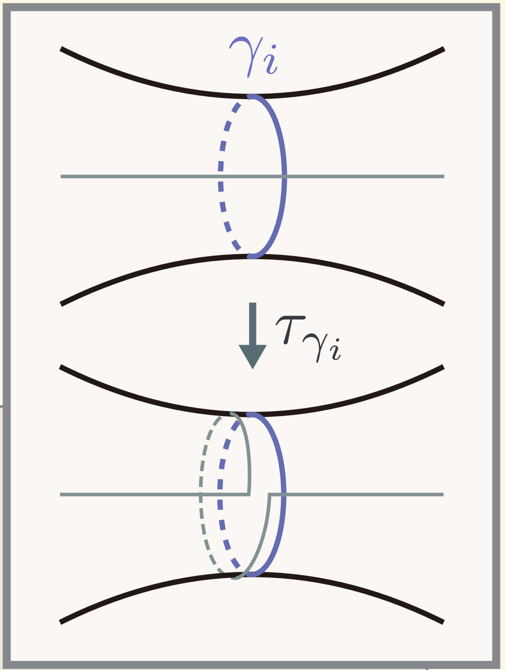
$\mathcal{A} \cup \mathcal{B}$ fills F

$\omega = \gamma_1\gamma_2\cdots\gamma_n$: (positive) word $\quad (\gamma_i \in \mathcal{A} \cup \mathcal{B})$

$\omega \rightsquigarrow \varphi_\omega = \varphi_{\gamma_1} \circ \varphi_{\gamma_2} \circ \cdots \circ \varphi_{\gamma_n} : F \to F$ defined by

$$\varphi_{\gamma_i} = \begin{cases} \tau_{\gamma_i}^{-1} & (\gamma_i \in \mathcal{A}) \\ \tau_{\gamma_i} & (\gamma_i \in \mathcal{B}) \end{cases},$$

where τ_{γ_i} : left-hand Dehn twist along γ_i.



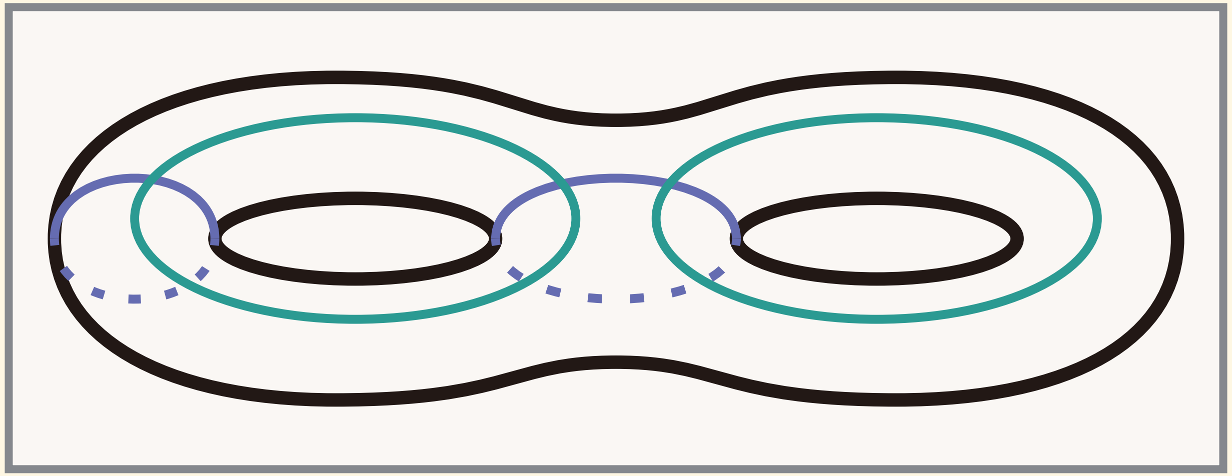
Theorem ([Penner, 1988])

Each α_i and β_i occur at least once in ω

$\Longrightarrow$ the class of φ_ω is pseudo-Anosov

Main Result

We have a complete combinatorial description of the veering triangulations of the mapping tori of the pA mapping classes arising from Penner's construction such that all complementary regions are not quadrilateral.



In this talk, we will describe the veering triangulation of the complement of the two-bridge knot $K[2, 2, 2, 2]$.

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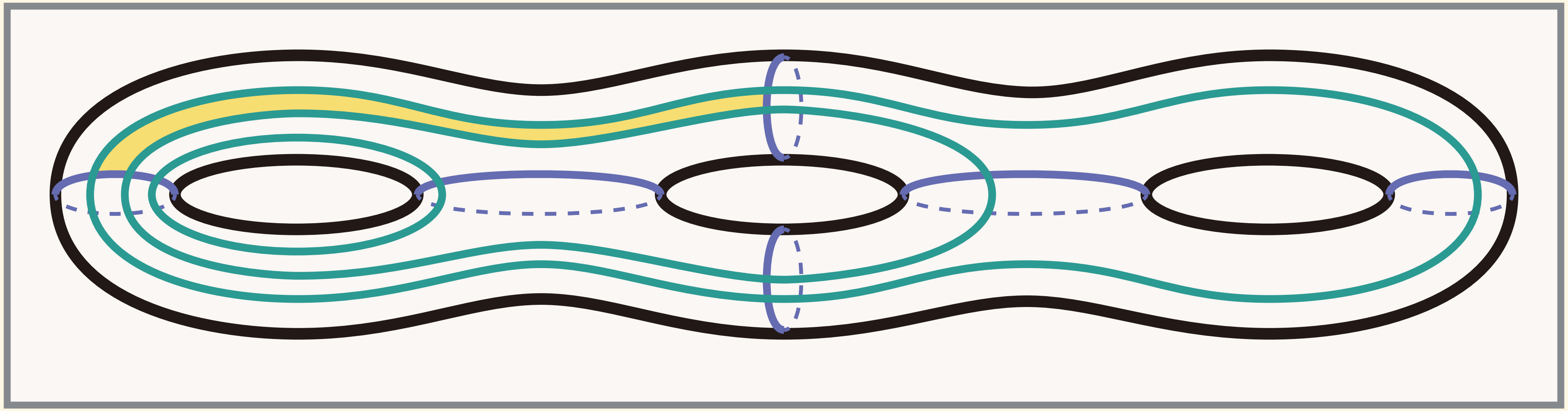


In this talk, we will describe the veering triangulation of the complement of the two-bridge knot $K[2, 2, 2, 2]$.

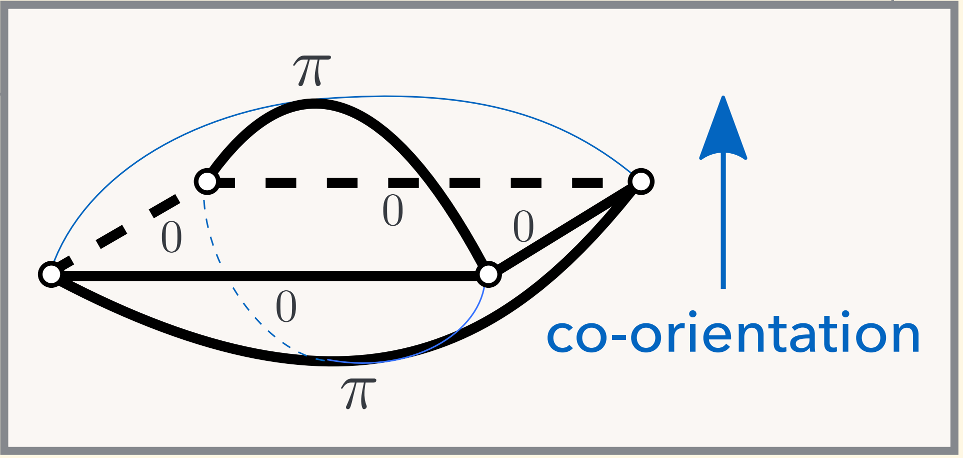
Taut structure (1)

Definition

an ideal tetrahedron: **taut**

$\overset{\text{def}}{\iff}$ (i) Each face is assigned a co-orientation so that two co-orientations point inwards and the others point outwards.

(ii) Each edge of the tetrahedron is assigned an angle of either π or 0 according to whether the co-orientations on the adjacent faces are the same or different.



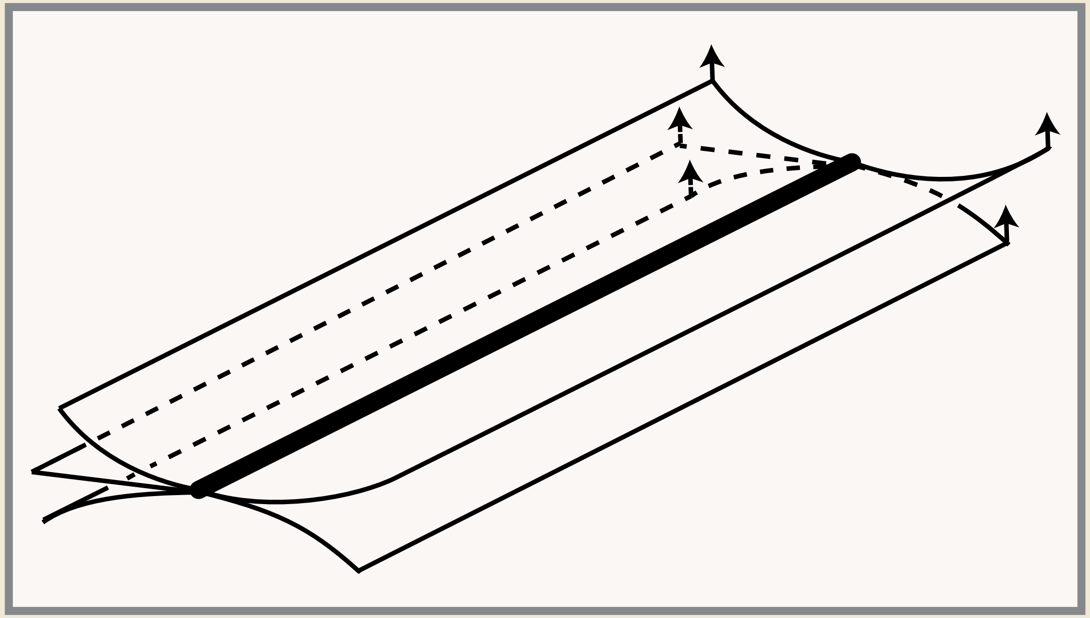
Taut structure (2)

Definition

An ideal triangulation of M_{φ° : **taut**

$\overset{\text{def}}{\Longleftrightarrow}$ (i) $\exists$a co-orientation assigned to each faces $\mathrm{s.t.}$ each ideal tetrahedron is taut.

(ii) The sum of the angles around each edge is 2π.

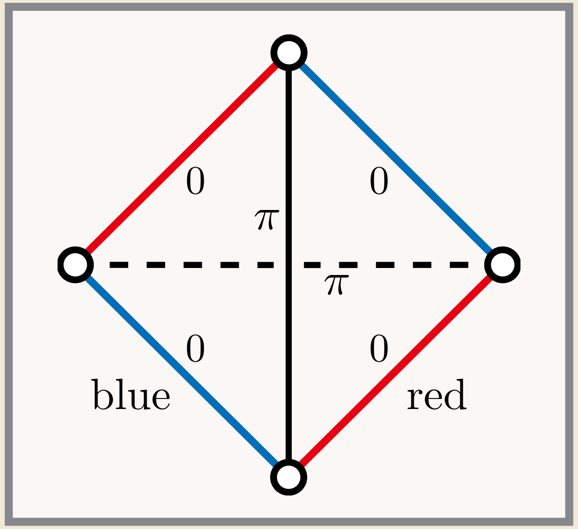


Veering triangulation

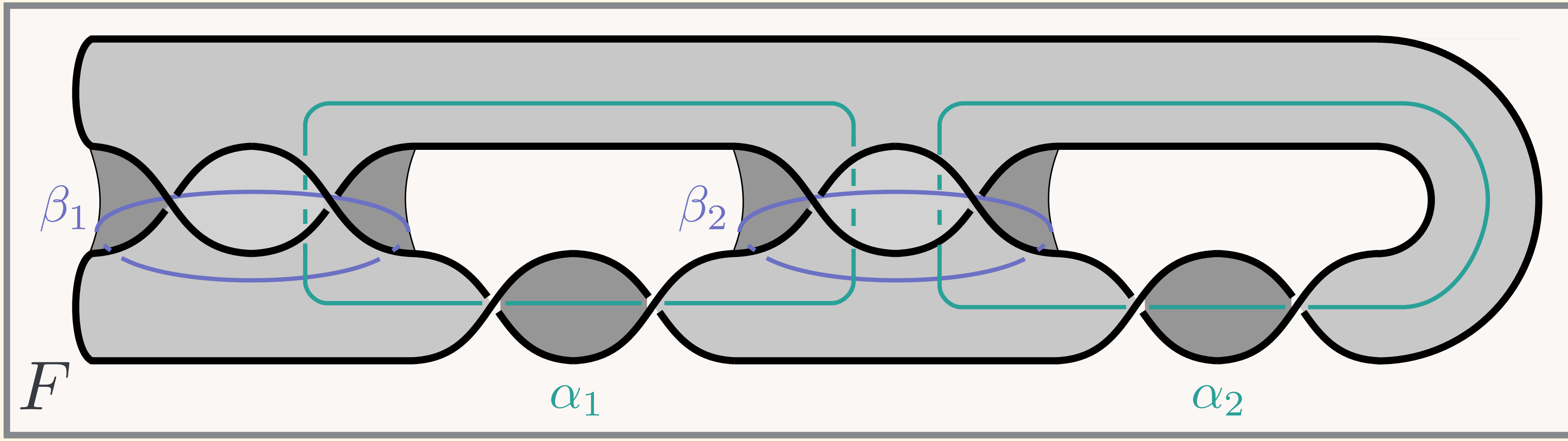
Definition

taut triangulation $\mathcal{D}$: **veering**

$\overset{\text{def}}{\Longleftrightarrow}$ $\exists$ assignment of two colors, red and blue, to all ideal edges of $\mathcal{D}$ so that every ideal tetrahedron can be sent by an orientation preserving homeomorphism to the following tetrahedron.



The monodromy of $K[2, 2, 2, 2]$



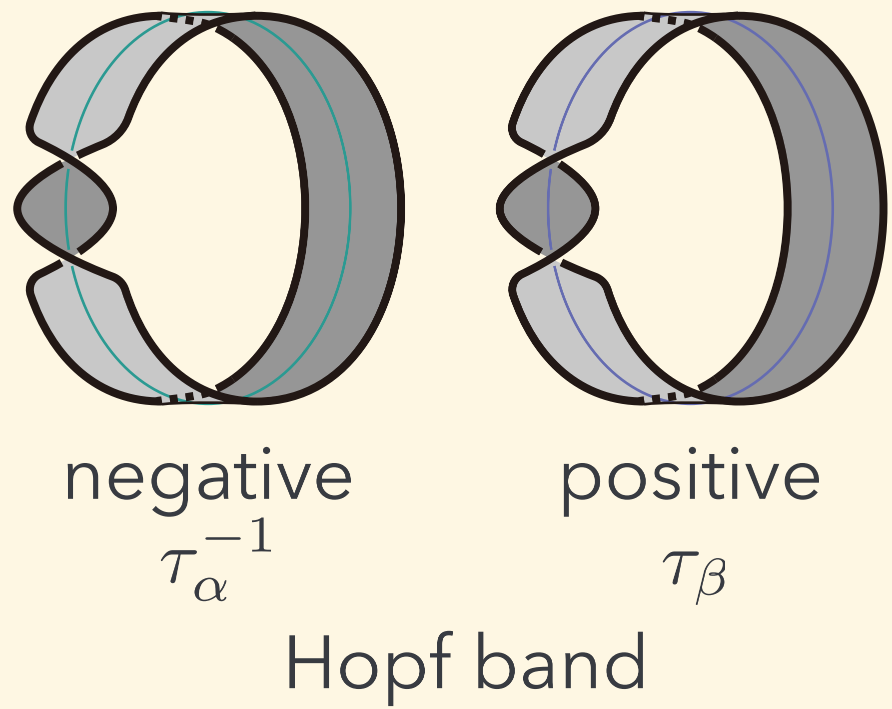
By [Gabai, 1988],
the monodromy φ of F is

$$\tau_{\alpha_1}^{-1} \circ \tau_{\alpha_2}^{-1} \circ \tau_{\beta_1} \circ \tau_{\beta_2}$$

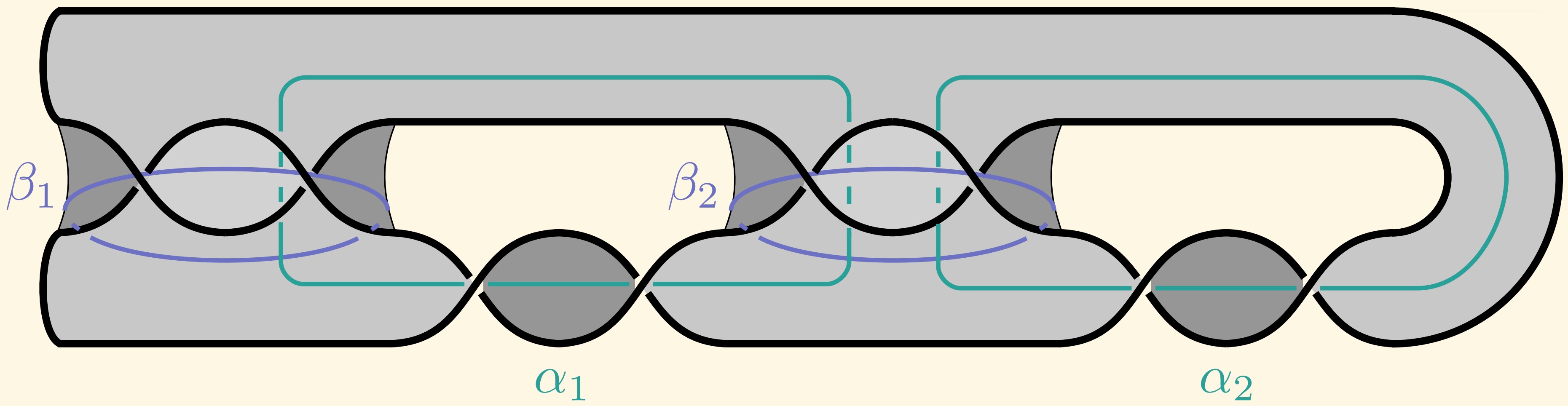
(τ_γ: left-hand Dehn twist along γ)

Remark

φ is a pA map arising from Penner's construction.

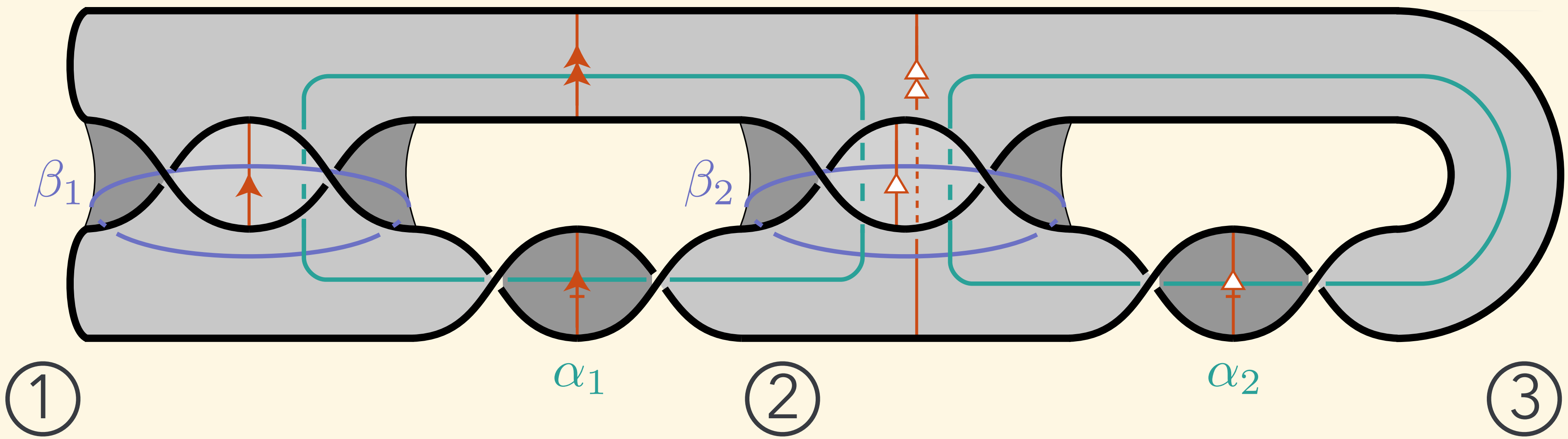


The veering triangulation of $K[2,2,2,2]$ (1)



At first, we give a triangulation of the surface F.
We consider a cell decomposition dual to the curves.

The veering triangulation of $K[2,2,2,2]$ (1)



Next, we cut F along the edges dual to the curves.
We add diagonals of the squares to the cell decomposition.

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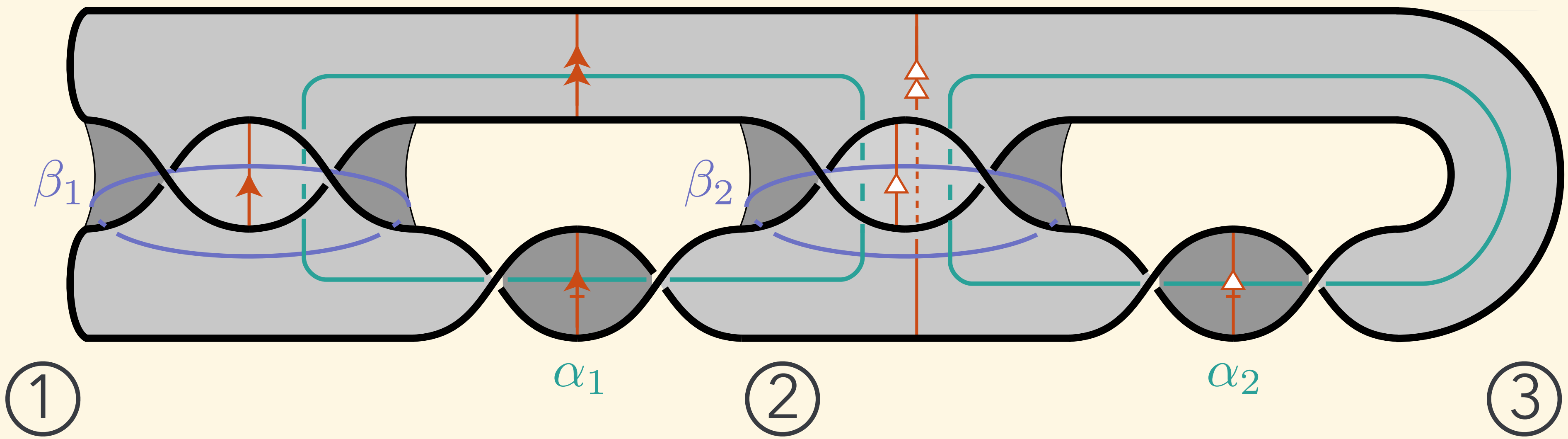

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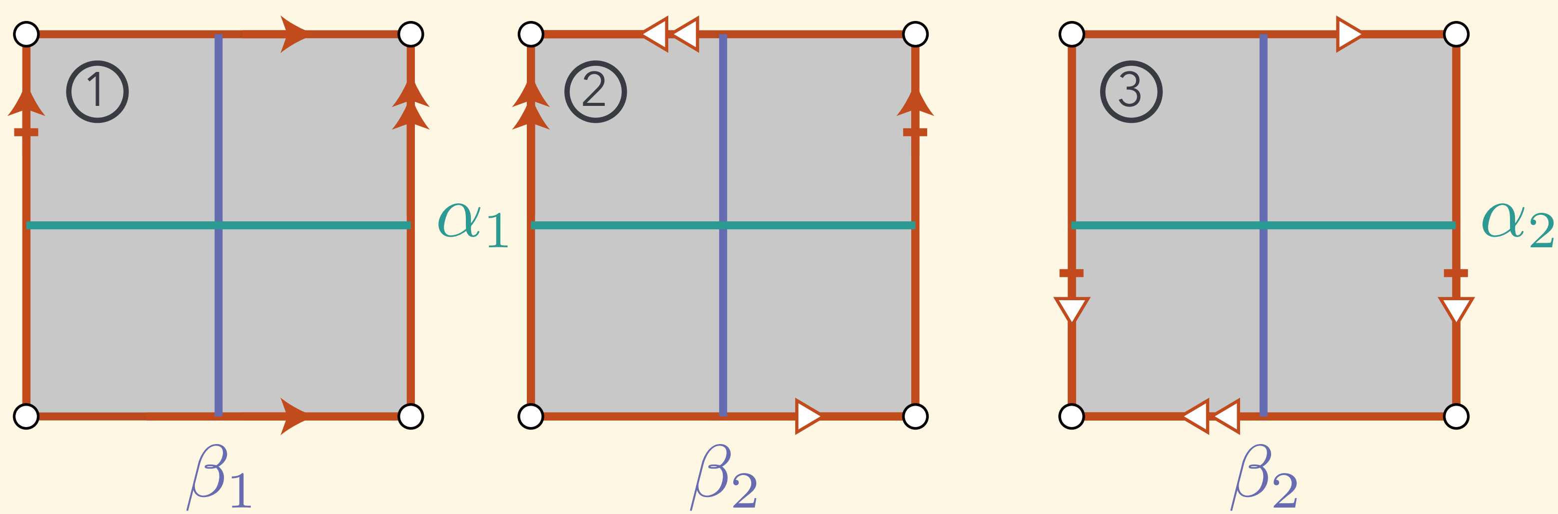

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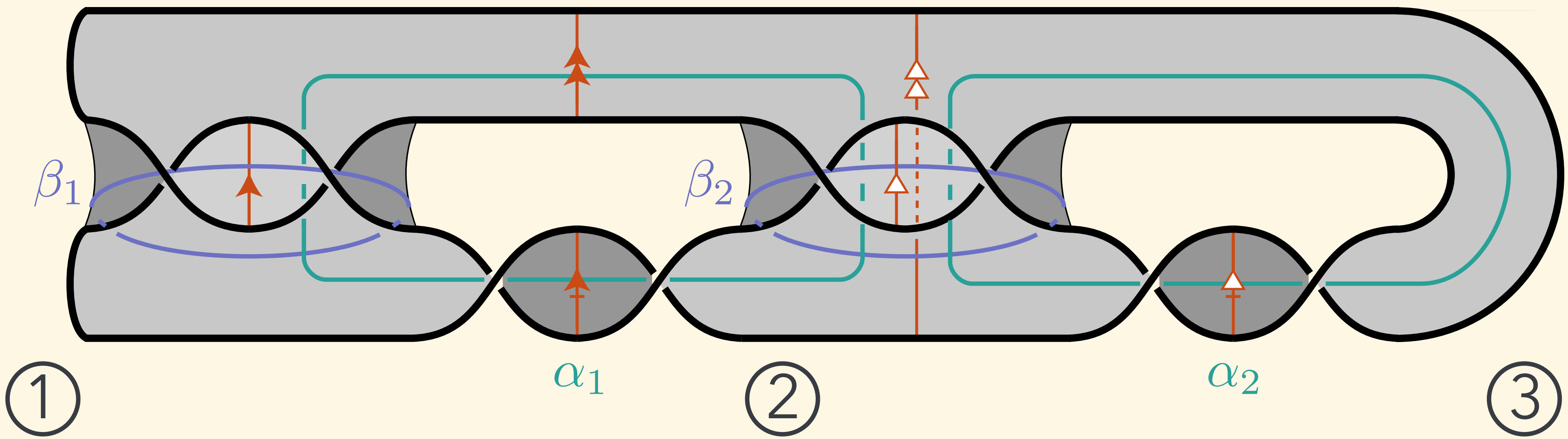

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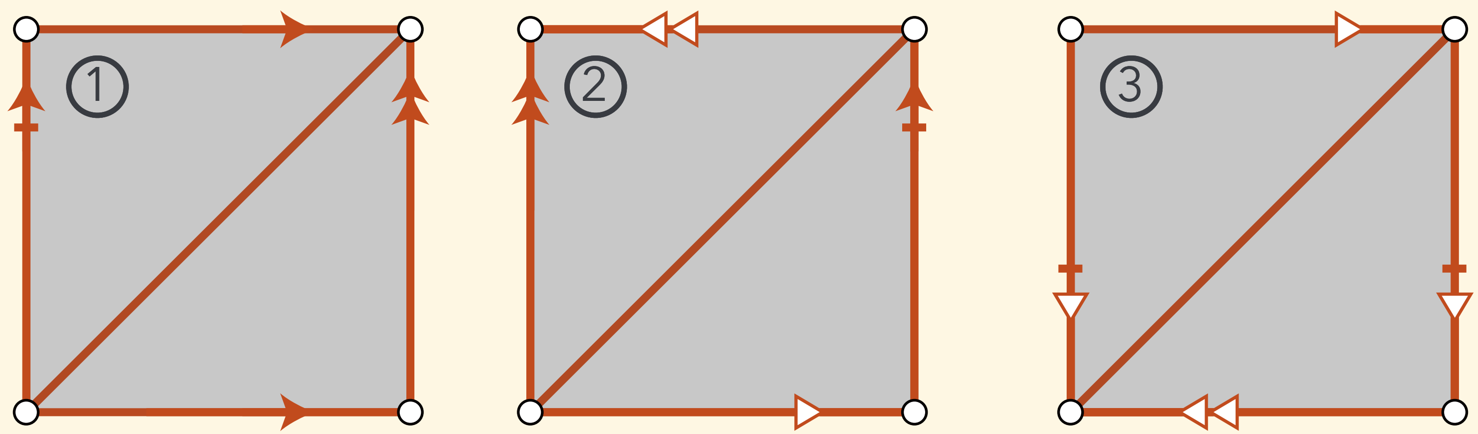
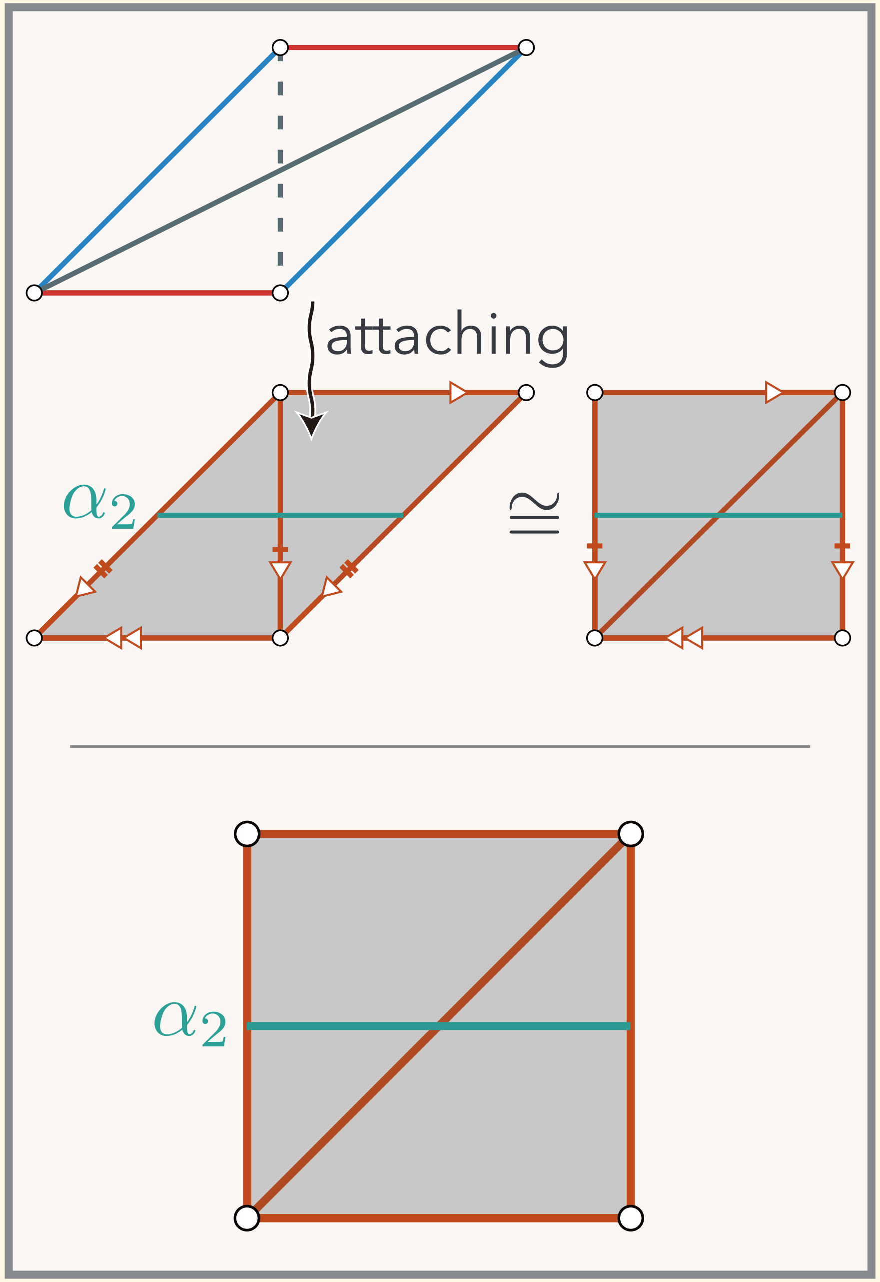


The veering triangulation of $K[2,2,2,2]$ (2)

We attach a veering tetrahedron to the triangulation along α_2.

Then there is a natural simplicial homeomorphism φ_{α_2} from the bottom annulus to the top annulus.

In fact, φ_{α_2} is the right-hand Dehn twist along α_2.



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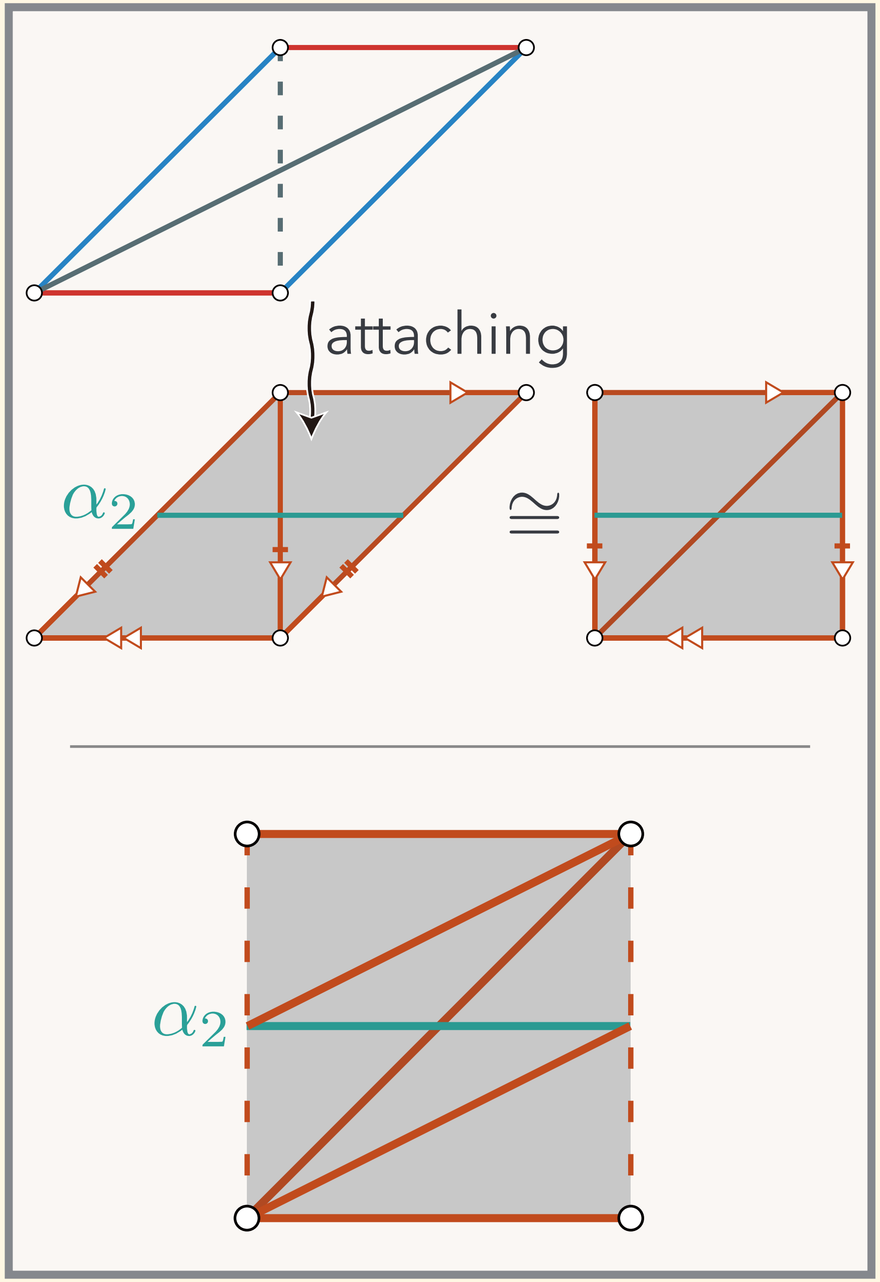
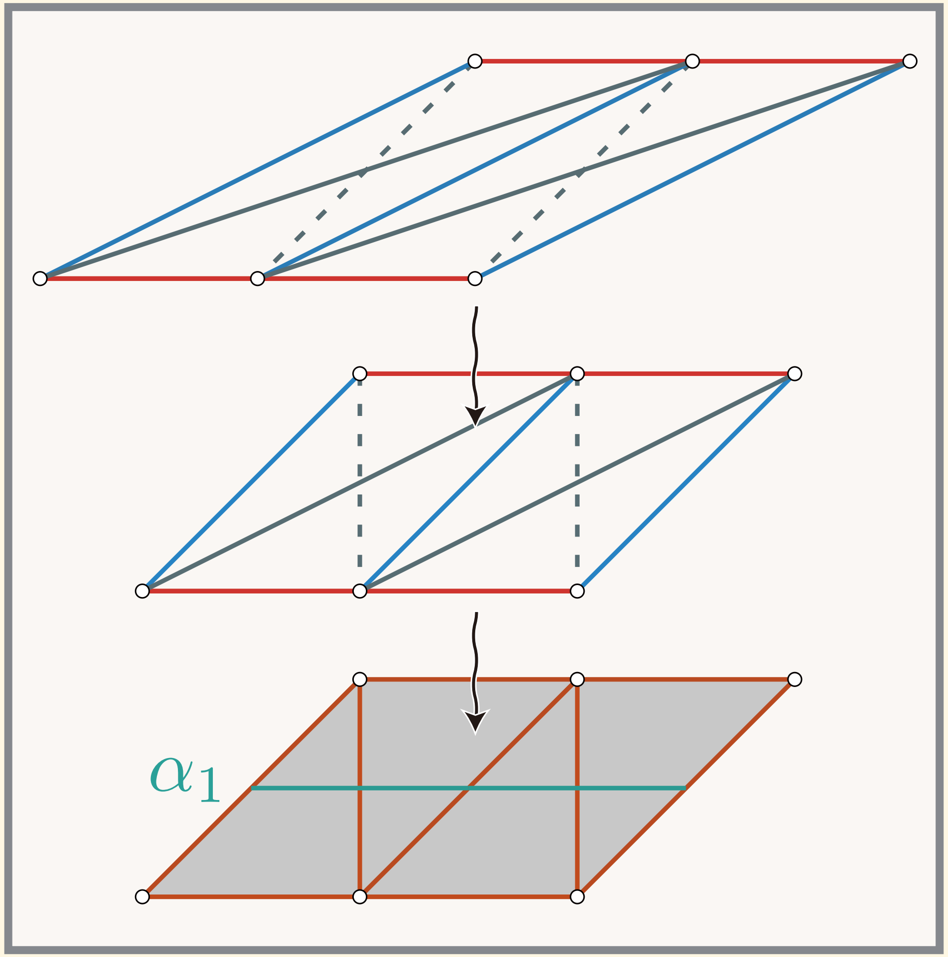


The veering triangulation of $K[2,2,2,2]$ (3)

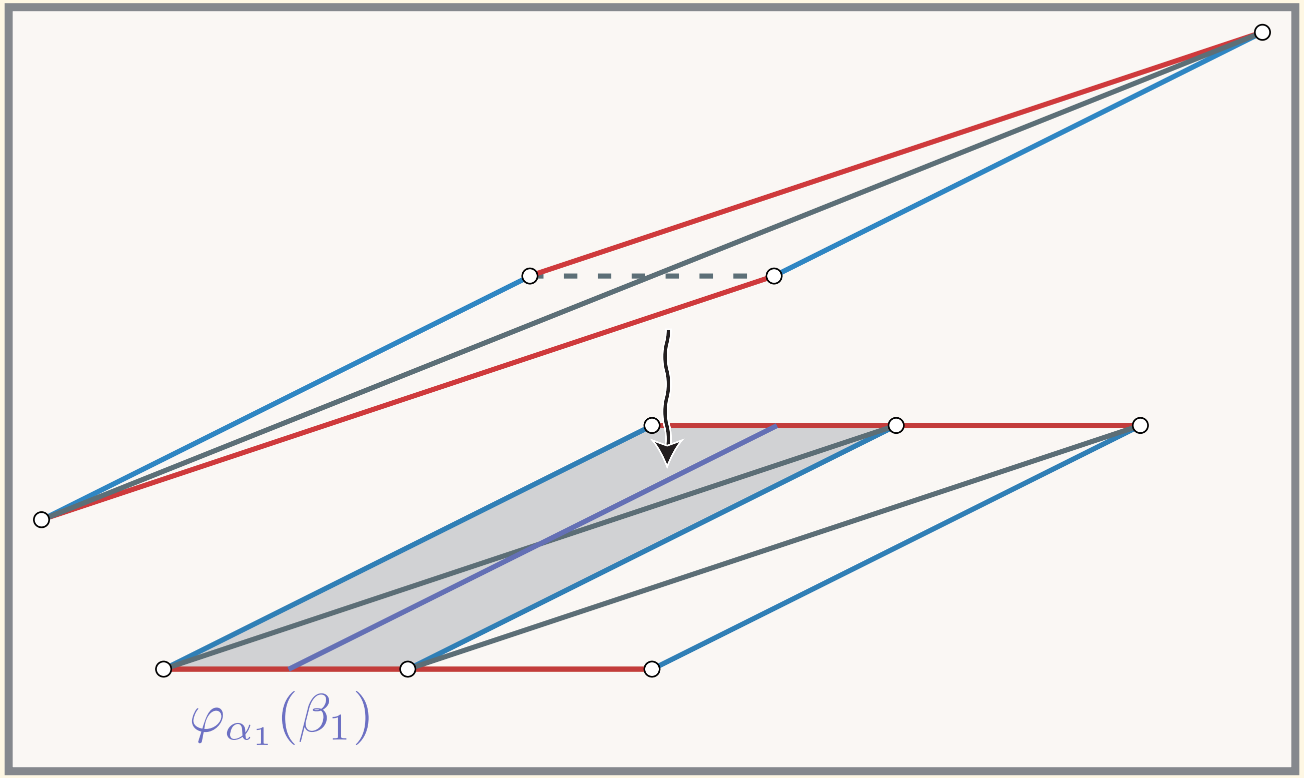
We also attach veering tetrahedra to the triangulation along α_1.

Then there is a natural simplicial homeomorphism φ_{α_1} from the bottom annulus to the top annulus.

The map φ_{α_1} is also the right-hand Dehn twist along α_1.



The veering triangulation of $K[2,2,2,2]$ (4)



We also attach veering tetrahedra to the triangulation along $\varphi_{\alpha_1}(\beta_1)$ and $\varphi_{\alpha_1} \circ \varphi_{\alpha_2}(\beta_2)$ because β_1 intersects α_1 and β_2 intersects α_1 and α_2.

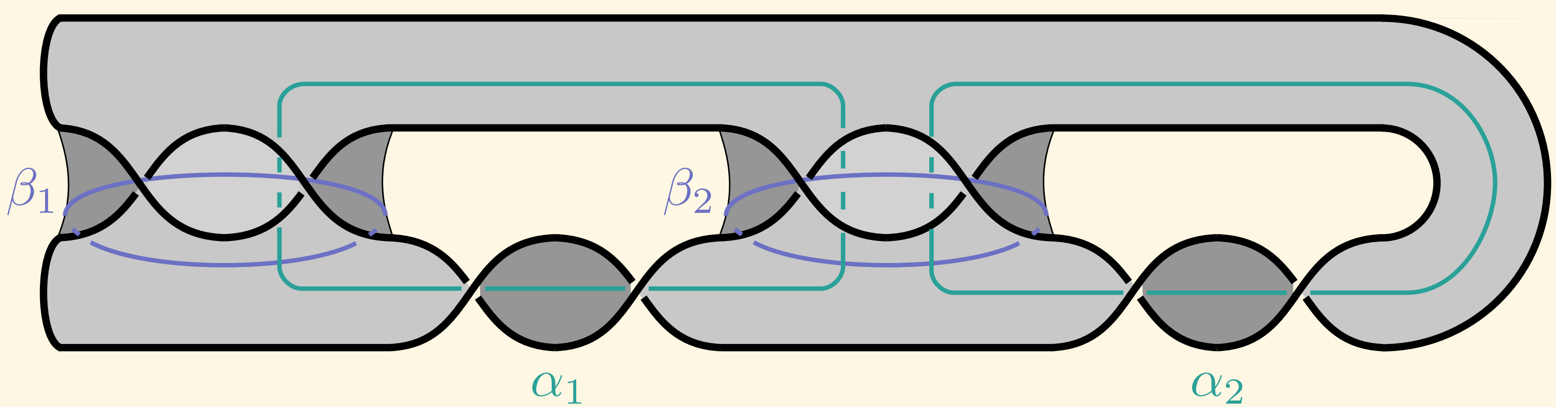
A homeo. φ_{β_1} (resp. φ_{β_2}) from the bottom to the top is the left-hand Dehn twist along $\varphi_{\alpha_1}(\beta_1)$ (resp. $\varphi_{\alpha_1} \circ \varphi_{\alpha_2}(\beta_2)$).

The veering triangulation of $K[2,2,2,2]$ (5)

We have an ideal triangulation of $F \times [0,1]$ s.t. each tetrahedron satisfies a condition for an ideal triangulation to be veering.

$$\begin{aligned}
&\varphi_{\beta_2} \circ \varphi_{\beta_1} \circ \varphi_{\alpha_2} \circ \varphi_{\alpha_1} : F \to F \\
&\quad = \left(\varphi_{\alpha_2} \circ \varphi_{\alpha_1} \circ \tau_{\beta_2} \circ \varphi_{\alpha_1}^{-1} \circ \varphi_{\alpha_2}^{-1}\right) \circ \left(\varphi_{\alpha_1} \circ \tau_{\beta_1} \circ \varphi_{\alpha_1}^{-1}\right) \circ \left(\tau_{\alpha_2}\right) \circ \left(\tau_{\alpha_1}\right) \\
&\quad = \left(\tau_{\alpha_2}^{-1} \circ \tau_{\alpha_1}^{-1} \circ \tau_{\beta_2} \circ \tau_{\alpha_1} \circ \tau_{\alpha_2}\right) \circ \left(\tau_{\alpha_1}^{-1} \circ \tau_{\beta_1} \circ \tau_{\alpha_1}\right) \circ \left(\tau_{\alpha_2}^{-1}\right) \circ \left(\tau_{\alpha_1}^{-1}\right) \\
&\quad = \tau_{\alpha_2}^{-1} \circ \tau_{\alpha_1}^{-1} \circ \tau_{\beta_2} \circ \tau_{\beta_1}
\end{aligned}$$

$= \varphi \leftarrow$ the monodromy of $K[2,2,2,2]$



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&\quad = \tau_{\alpha_2}^{-1} \circ \tau_{\alpha_1}^{-1} \circ \tau_{\beta_2} \circ \tau_{\beta_1}
\end{aligned}$$

$= \varphi$ ← the monodromy of $K[2,2,2,2]$

Hence, we have the veering triangulation of $K[2,2,2,2]$.

Future work (1)

By using similar argument, we can describe the veering triangulation of the mapping torus of each pA mapping class arising from Penner's construction s.t. all complementary regions are not quadrilateral.

The above construction assumes that each complementary region has a singular point of the stable/unstable foliation. However, a quadrilateral region does not have a singular point.

Question

How can we describe the veering triangulation when some complementary regions are quadrilateral?

Future work (2)

Recall (Main Question)

Is the veering triangulation of the mapping torus of each pA mapping class arising from Penner's construction geometric?

↓

Can we apply Casson-Rivin volume maximization theory?

Guéritaud has proved that the veering triangulation of each once-punctured torus bundle over S^1 is geometric by using the theory.

Future work (2)

Recall (Main Question)

Is the veering triangulation of the mapping torus of each pA mapping class arising from Penner's construction geometric?

↓

Can we apply Casson-Rivin volume maximization theory?

Guéritaud has proved that the veering triangulation of each once-punctured torus bundle over S^1 is geometric by using the theory.

Thank you for your attention!

By using similar argument, we can describe the veering triangulation of the mapping torus of each pA mapping class arising from Penner's construction s.t. all complementary regions are not quadrilateral.

$\omega = \gamma_1\gamma_2\cdots\gamma_n$: (positive) word $(\gamma_i \in \mathcal{A} \cup \mathcal{B})$

Step 1:

Construct an ideal triangulation of the surface F like the previous example.

Step 2: (repeat)

Attach m_i^2 veering tetrahedra to the top ideal triangulation along $\varphi_{i-1} \circ \cdots \circ \varphi_1(\gamma_i)$, where $m_i = \#(\gamma_i \cap (\mathcal{A} \cup \mathcal{B}))$. Then, we have a natural simplicial homeomorphism φ_i.

Step 3:

Glue the bottom and the top of $F \times [0,1]$ by $\varphi_n \circ \cdots \circ \varphi_1$.