By Dehn surgery, we mean an operation to create a new 3-manifold from a given one and a given knot in it as follows: Take an open tubular neighborhood of the knot, remove it, and glue a solid torus back.

A motivation to study Dehn surgery comes from the following famous fact, now called Hyperbolic Dehn Surgery Theorem, due to W.P. Thurston [4]: On a hyperbolic knot (i.e., a knot with hyperbolic complement), all but finitely many Dehn surgeries yield hyperbolic 3-manifolds. In view of this, such finitely many exceptions are called exceptional surgeries. Then it is natural to ask: How many exceptional surgeries can occur on each knot? Concerning this question, C.McA. Gordon conjectured that:

**Conjecture** ([2, Problem 1.77]). There exist at most 10 exceptional surgeries on each hyperbolic knot.

Recall that Dehn surgery on a knot $K$ is called surgery along the slope $\gamma$ if the curve identified with the meridian of the attached solid torus via the surgery represents $\gamma$ on the peripheral torus of $K$. Then our first result is the following;

**Theorem 1** ([1]). Let $\mu$ be any slope for a hyperbolic knot $K$. Then there are at most 10 exceptional surgeries on $K$ along slope $\gamma$ with $\Delta(\mu, \gamma) \leq 1$.

Here the distance $\Delta(\gamma_1, \gamma_2)$ between two slopes $\gamma_1, \gamma_2$ is defined as the minimal intersection number between the representatives of the slopes.

When $K$ is a knot in the 3-sphere $S^3$, by using the standard meridian-longitude system, slopes on the peripheral torus of $K$ are parametrized by rational numbers with 1/0. See [3] for example. Then the Dehn surgery on $K$ along the meridional slope 1/0 is called the trivial Dehn surgery on $K$ in $S^3$. It yields $S^3$ again, which
is obviously exceptional if $K$ is hyperbolic. We say that a Dehn surgery on $K$ in $S^3$ is integral if it is along a slope corresponding an integer. This means that the slope is represented by a curve which runs longitudinally once.

Thus we have the following corollary from Theorem 1.

**Corollary 2 ([1]).** On any hyperbolic knot in $S^3$, there are at most 9 non-trivial integral exceptional surgeries.

On the other hand, we obtain the next independently;

**Theorem 3.** On a hyperbolic alternating knot in $S^3$, non-trivial exceptional surgeries are all integral.

From Theorem 3 together with Corollary 2, it follows that:

**Theorem 4.** On a hyperbolic alternating knot in $S^3$, there are at most 10 exceptional surgeries.

Therefore the Gordon’s conjecture is true for such knots.

**References**


