On exceptional surgeries on Montesinos knots

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Dehn surgery on a knot

- $K$: a knot in $S^3$
- $E(K)$: the exterior of $K$ (i.e., $S^3 \setminus $ (open tubular nbd. of $K$))

**Dehn surgery**: Gluing a solid torus to $E(K)$

$\gamma = [f(m)]$: surgery slope, identified with $r \in \mathbb{Q} \cup \{1/0\}$.

$K(r)$: the manifold obtained by Dehn surgery on $K$ along $\gamma = r$.

**Theorem [Lickorish, Wallace]**

Every pair of closed orientable 3-manifolds are related by a finite sequence of Dehn surgeries.
Exceptional surgery

Dehn surgery on a hyperbolic knot yielding a non-hyperbolic mfd.

Theorem [Thurston]

Exceptional surgeries are only finitely many for each hyperbolic knot.

Each exceptional surgery is either:

- **Reducible surgery** (yielding a mfd. containing an essential $S^2$)
- **Toroidal surgery** (yielding a mfd. containing an essential $T^2$)
- **Seifert surgery** (yielding a Seifert fibered mfd.)

as a consequence of the Geometrization Conjecture established by Perelman (2002-03).
Montesinos knot $M(R_1, \ldots, R_l)$

A knot admitting a diagram obtained by putting rational tangles $R_1, \ldots, R_l$ together in a circle.

$M(\frac{1}{2}, \frac{1}{3}, -\frac{2}{3})$

- **length** of the knot $=$ minimal number of rational tangles.
- $P(a_1, \cdots, a_n) = M(\frac{1}{a_1}, \cdots, \frac{1}{a_n}) : (a_1, \cdots, a_n)$-pretzel knot.
Our problem

**Problem**

Determine and classify exceptional surgeries on hyperbolic Montesinos knots.

**Remark [Menasco], [Oertel], [Bonahon-Siebenmann]**

Non-hyperbolic Montesinos knots are $T(2, n)$ and, $P(-2, 3, 3)(=T(3, 4))$, $P(-2, 3, 5)(=T(3, 5))$.

$T(x, y)$ : the $(x, y)$-torus knot.

**Remark**

Dehn surgeries on the torus knots have been completely classified by Moser.
Known facts: Length other than 3

\(K\) : hyperbolic Montesinos knot with length \(l\)

- \(l \leq 2 \implies K\) is a two-bridge knot.
  - Exceptional surgeries for them are completely classified [Brittenham-Wu].
- \(l \geq 4 \implies K\) admits no exceptional surgery [Wu].

Remains

Exceptional surgeries on \(M(R_1, R_2, R_3)\) (i.e. \(l = 3\))
Known facts: Reducible / Toroidal surgery

- \( \exists \) reducible surgeries on Montesinos knots [Wu].
- Toroidal surgeries on Montesinos knots are completely classified [Wu].

Remains

Seifert surgeries on \( M(R_1, R_2, R_3) \)
Known facts: Toroidal Seifert surgery

Recall: Each exceptional surgery is either:

- Reducible,
- Toroidal,
- Seifert.

Remark [Eudave-Muñoz]
They are not exclusive. (i.e., there are non-empty intersection.)

Theorem [Motegi]
A knot with $|Sym^*(K)| > 2$ admits no toroidal Seifert surgery.
In particular, other than the trefoil knot, no two-bridge knots admit toroidal Seifert surgeries.
Results: Toroidal Seifert surgery

**Theorem [Ichihara-J.]**
Montesinos knots admit no toroidal Seifert surgeries other than the trefoil knot.

**Corollary**
A hyperbolic Montesinos knot admits no toroidal Seifert surgery.

**Remains**
Atoroidal Seifert surgeries on $M(R_1, R_2, R_3)$ (i.e. yielding a Seifert mfd. over $S^2$ with $\leq 3$ exceptional fibers)
Known facts: $\pi_1(K(r))$ is cyclic / Finite
($K(r)$ is atoroidal Seifert fibered)

**Theorem [Ichihara-J.]**

$K$: hyperbolic Montesinos knot

- If $\pi_1(K(r))$ is cyclic, then $K = P(-2, 3, 7)$ and $r = 18$ or $19$.
- If $\pi_1(K(r))$ is acyclic finite, then $K = P(-2, 3, 7)$ and $r = 17$, or $K = P(-2, 3, 9)$ and $r = 22$ or $23$.

**Remains**

Atoroidal Seifert surgeries on $K = M(R_1, R_2, R_3)$ with
$|\pi_1(K(r))| = \infty$
(i.e. yielding a Seifert mfd. over $S^2(n_1, n_2, n_3)$ with
$$\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} \leq 1$$)
Alternating knots

An **alternating diagram** = the crossings alternate under, over, under, over, as you travel along the knot.

A knot is **alternating** if it has an alternating diagram.

\[ P(3,5,8) = \]

\[ P(-3,5,8) = \]

**Remark**

\[ M(R_1, \ldots, R_l) : \text{alternating} \iff R_1, \ldots, R_l \text{ have the same sign.} \]
Results: atoroidal Seifert surgery

**Theorem [Ichihara-J.-Mizushima]**

If $K = M(R_1, R_2, R_3)$ with $R_1, R_2, R_3 > 0$ (i.e. $K$ is alternating) admits an atoroidal Seifert surgery, then $K = P(a, b, c)$ with odd integers $3 \leq a < b < c$.

**Theorem [Wu]**

If $M\left(\frac{p_1}{q_1}, \frac{p_2}{q_2}, \frac{p_3}{q_3}\right)$ with $q_1 \leq q_2 \leq q_3$ admits an atoroidal Seifert surgery, then $q_1 = 2$, $(q_1, q_2) = (3, 3)$, or $(q_1, q_2, q_3) = (3, 4, 5)$.

**Corollary**

An alternating hyperbolic Montesinos knot with length 3 admits no Seifert surgery.
Today’s aim

We will show the following.

**Proposition**

Let $K = P(2n + 1, 2m + 1, 2m + 1)$ with $n, m \in \mathbb{N}$. Then $K$ admits no atoroidal Seifert surgery.

Key tools:
- the double branched covering space of a knot
- the Rasmussen invariant and the signature
- the alternation number