Chirally cosmetic fillings on a hyperbolic manifold

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joint work with
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§1. Definition of cosmetic fillings

§2. Examples and Conjectures

§3. Main Result (which gives a counterexample of a conjecture)

§4. Construction of Main Result
§1. Definitions

$X$: an oriented compact 3-mfd. with $\partial X = T^2$

$X(r)$: the 3-mfd. obtained by $r$-Dehn filling on $X$ ($r \in \mathbb{Q} \cup \{1/0\}$)

**Definition (cosmetic filling)**

$r_1$-filling and $r_2$-filling on $X$ are **cosmetic**

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- Cosmetic fillings are **purely** (resp. **chirally**)
  $$\iff h \text{ is orientation preserving (resp. reversing).}$$

- Cosmetic fillings are **mundane**
  $$\iff \exists \varphi: X \to X: \text{homeo. s.t. } \varphi(r_1) = r_2.$$ 

- Cosmetic fillings are **exotic** $$\iff \text{These are not mundane.}$$
### Example (mundane cosmetic fillings)

- **$U$:** the unknot in $S^3$, $n \in \mathbb{Z}$
  
  $\Rightarrow \frac{1}{n}$- and $1/0$-filling on $E(U)$ are truly mundane cosmetic.

  
  \[
  \frac{1}{n} \quad \cong \quad \frac{1}{0}
  \]

- **$K$:** an amphicheiral knot in $S^3$, $r \in \mathbb{Q} \setminus \{0\}$
  
  $\Rightarrow (\pm r)$-fillings on $E(K)$ are chirally mundane cosmetic.
§2. Examples and Conjectures

Example (mundane cosmetic fillings)

- \( U \): the unknot in \( S^3 \), \( n \in \mathbb{Z} \)
  \[ \Rightarrow 1/n \text{-} \text{and} \ 1/0 \text{-} \text{filling on } E(U) \text{ are truely mundane cosmetic.} \]
  \[ \frac{1}{n} \text{ } \approx \text{ } \frac{1}{0} \]

- \( K \): an amphicheiral knot in \( S^3 \), \( r \in \mathbb{Q} \setminus \{0\} \)
  \[ \Rightarrow (\pm r) \text{-} \text{fillings on } E(K) \text{ are chirally mundane cosmetic.} \]

Interests: **exotic** cosmetic fillings

**Cosmetic Surgery Conjecture**

Cosmetic fillings are never purely and exotic.

Today’s target: **chirally and exotic** cosmetic fillings
Example [Bleiler-Hodgson-Weeks]

∃ a hyperbolic knot $K \subset S^2 \times S^1$ s.t. $E(K)$ admits chirally exotic cosmetic fillings yielding $L(49, \pm 18)$. ($L(49, -18) = L(49, 19)$)

Conjecture [Bleiler-Hodgson-Weeks]

Cusped hyperbolic manifolds admit no exotic cosmetic fillings, truly or chirally, yielding hyperbolic mfds.
§2. Examples and Conjectures

Example [Bleiler-Hodgson-Weeks]

\[ \exists \text{ a hyperbolic knot } K \subset S^2 \times S^1 \text{ s.t. } E(K) \text{ admits chirally exotic cosmetic fillings yielding } L(49, \pm 18). \quad (L(49, -18) = L(49, 19)) \]

Conjecture [Bleiler-Hodgson-Weeks]

Cusped hyperbolic manifolds admit no exotic cosmetic fillings, truly or chirally, yielding hyperbolic mfds.

We give a counterexample for the chirally case.
§3. Main Result

Main Theorem [Ichihara-J.]

\[ \exists \text{a hyperbolic manifold admitting exotic chirally cosmetic fillings which yield hyperbolic manifolds.} \]
§4. Construction of Main Result

$L$: a link in $S^3$, \hspace{1cm} $L'$: a link obtained from $L$ by a banding

**Definition (chirally cosmetic banding)**

A banding is said to be **chirally cosmetic** if
\[ \iff \exists h: S^3 \to S^3 : \text{ori. reversing homeo. s.t. } h(L) = L'. \]

**Lemma ("Montesinos trick")**

$K$: a knot in $S^3$ admitting a **chirally cosmetic** banding

$\Sigma_K$: the double branched cover of $S^3$ branched along $K$

\[ \implies \] We have **chirally cosmetic fillings** on the exterior of a knot in $\Sigma_K$. 
Proposition [Ichihara-J.]

$9_{27} = S(49, 18)$ admits a cosmetic banding as follows:

Remark

- This is obtained from the cosmetic fillings of [B.-H.-W.].
- $9_{27} = C(1, 1, 1, 2, -1, -1, -1, -2) = C(2, 2, -2, 2, 2, -2)$
- $9_{27}$ is a symmetric union of $5_2$. In particular, it is ribbon.
§4. Construction of Main Result

$9_{27}$ can be obtained as follows:
§4. Construction of Main Result

This yields many chirally cosmetic banding by adding twists of the tangles, or increasing the number of tangles from $3 \times 2$ to $n \times 2$, ...
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The followings are shown by using computer programs SnapPy and hikmot.

- $\Sigma_K$ and $\Sigma_K \setminus \bar{K}$ are hyperbolic.
- The slopes 3 and 1/0 on $\partial N(\bar{K})$ in $\Sigma_K$ are inequivalent.
  (i.e. these cosmetic fillings are exotic.)