Strong cylindricality and the monodromy of bundles

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Essential surfaces

have played a very important role in the study of 3-manifolds.

Definition

\[ F : \text{connected closed surface, not homeomorphic to } S^2, \text{ embedded in a 3-manifold } M \]

\[ F : \text{essential } \iff F \text{ is incompressible & not-}\partial\text{-parallel} \]
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Essential surfaces always exist in \( M \) if \( \beta_1(M) \geq 1 \), and are actually **infinitely many** up to isotopy if \( \beta_1(M) \geq 2 \).
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**Finiteness result (Hass, ’95)**

Any closed hyperbolic 3-manifold contains only **finitely many acylindrical** essential surfaces.

**Remark:** acylindrical = not cylindrical
Cylindrical surface

Definition

\[ F : \text{cylindrical} \Leftrightarrow M - \text{int} N(F) \supset \text{essential annulus } \mathcal{A} \]
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**Fact (Hass, ’95)**

In a closed hyperbolic 3-manifold, any essential surface of **sufficiently large genus** is cylindrical.
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**Fact (Hass, ’95)**

In a closed hyperbolic 3-manifold, any essential surface of **sufficiently large genus** is cylindrical.

**Fact (Eudave-Muñoz–Neumann-Coto, ’04)**

In a 3-manifold with triangulation of \( t \) tetrahedra, any essential surface of genus \( g \geq t + 1 \) is cylindrical.
Strong cylindricality & Theorem

Definition

$F$ is strongly cylindrical

$\Leftrightarrow F$ is cylindrical with $(A, \partial A) \subset (M, F)$, embedded.
**Strong cylindricality & Theorem**

**Definition**

\[ F \text{ is strongly cylindrical} \iff F \text{ is cylindrical with } (A, \partial A) \subset (M, F), \text{ embedded.} \]

**Fact (Schleimer, 03')**

In a 3-manifold with triangulation of \( t \) tetrahedra, any essential surface of genus \( g \gg t \) is strongly cylindrical.

Thus, any connected hyperbolic 3-manifold contains only finitely many weakly acylindrical surfaces.

(weakly acylindrical = not strongly cylindrical)
Strong cylindricality & Theorem

**Definition**

$F$ is **strongly cylindrical**

$\iff F$ is cylindrical with $(A, \partial A) \subset (M, F)$, embedded.

**Fact (Schleimer, 03’)**

In a 3-manifold with triangulation of $t$ tetrahedra, any essential surface of genus $g \gg t$ is strongly cylindrical.

Thus, any connected hyperbolic 3-manifold contains only **finitely many weakly acylindrical** surfaces.

*(weakly acylindrical $=$ not strongly cylindrical)*

**Theorem [I.-Kobayashi-Rieck]**

$M$: connected 3-manifold with triangulation of $t$ tetrahedra.

$F$: connected essential surface of genus $g$.

Then \( g \geq 38t \implies F \) is strongly cylindrical.
Surface bundle and Monodromy (Motivation)

\( M \): surface bundle over \( S^1 \) with fiber \( F \)

i.e., \( M \cong F \times [0, 1]/\varphi \) with \( \varphi : F \to F \), monodromy
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When \( F \) is strongly cylindrical, by isotoping the annulus \( A \), we can find an essential loop \( \gamma \) on \( F \) such that \( \gamma \cap \varphi(\gamma) = \emptyset \).
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\[ \implies \text{The action of } \varphi \text{ on the curve complex of } F \]
has the translation distance at most 1.

Corollary [Schleimer, I.-Kobayashi-Rieck]

Suppose that a closed hyperbolic manifold \( M \) admits infinitely many fibrations over \( S^1 \) with connected fibers. Then all but finitely many fibrations on \( M \) have translation distance 1.